Non-uniform Random Links in Small-world Graphs: Models, Analysis and Applications in Network Designs *

Van Nguyen and Charles Martel
{nguyenvk,martel}@cs.ucdavis.edu
Computer Science, UC Davis, CA 95616

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Abstract

Four decades have gone since the first experiment (Milgram, 1967 [3]) which confirms a (probably, centuries old) folklore that we are in a small-world where any two strangers can be linked by a short chain of acquaintances. The recent extensive multi-disciplinary research initiated by Watts and Strogatz's seminal work (1998 [4]) shows that small-world properties are seen common in several large-scale real-world networks such as in social networks and the Internet networks. Yet, this striking phenomenon was not fully understood (and contemplated) till recently when Kleinberg (2000,[1]) emphasizes and produces a nice model for the other striking aspect that such a short chain can be found using limited local information only (e.g. a search based on a first-name basis).

Starting with Watts and Strogatz, much work has been done recently on models where random links are added to a simple graph which is rich in local contacts. While these local contacts are likely the source for high clustering, the random links (uniform in early models) make the bridging long-range contacts which shrink the graph diameter. Kleinberg, however, uses non-uniform random links, which favor the closer nodes over the distant ones. Successful with some special distribution of random links, his work introduces a new "algorithmic perspective" on this research field and initiates a new active branch on decentralized search (in small-worlds) which could be desired in many Internet-related scenarios such as in peer-to-peer networking. Inspired by these results, we, however, envision a potential, new direction of research on random structures, adding to the classical study of random graphs, and that this new approach can contribute to network designs by introducing constructions which simultaneously optimize many practical factors.

We are interested in a more general study of the random structures which are formed by adding a generic distribution (often non-uniform) of random link to a generic local-contact graph. We desire to learn general rules and characteristics in making this formation feature small-world (and related) properties and also consider to use our observations to design practical networks. Particularly, we consider the abstract properties of the random link distributions which can introduce shortcuts (typically, with length as a poly-log of size) between the sites of the local-contact base graph. By finding such general properties, we come up with a *thorough analysis of Kleinberg's small-world models* [1, 2]. Moreover, we develop a general framework to construct small-world graphs, featuring a hierarchical family of classes of random structures where *short paths are available and yet can be found using decentralized routing strategies in our more refined classes*. We suggest *routing strategies (and routing data structures) which could best exploit the special features of our network constructions*, based on which, we consider to simultaneously optimize many network factors such as construction cost, expected route length and congestion. Different to classical random graphs, non-uniformity creates our main technical challenge here; thus, we also contribute in developing techniques for analyzing structures with non-uniform random links.

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References

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